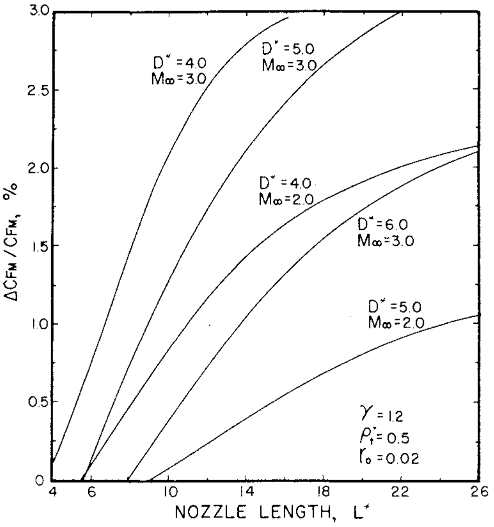

Fig. 3 Effect of vehicle size on performance.

it is possible that the flow could separate from the nozzle contour.

An analysis of the terms in Eq. (1) shows that expanding to lower pressures decreases the pressure thrust and base thrust. Base thrust decreases are a direct result of the coupling between r and r_b (p_s and p_b), since r_b decreases as r decreases. Divergence losses also increase, since larger cone angles are required to obtain larger area ratios with fixed nozzle length. However, the momentum thrust increases due to the increase in velocity brought about by the further expansion. It is obvious that the maximum thrust occurs at some point of trade-off between the above terms.

Figure 2 is a plot of Eq. (2) for a particular set of operating conditions, illustrating the thrust maximums discussed above. Also shown is the locus of $(C_F)_{MAX}$ determined from the design condition, Eq. (5). Long nozzles ($L^* > 12$) do not exhibit a thrust maximum except at the limiting case of no base area (i.e., $\epsilon = 16$). For a fixed L^*, the performance peak occurs at the trade-off point mentioned above, except for the long nozzles which completely fill the vehicle base. Similar trends were exhibited at different flight M_∞'s and vehicle area ratios (proportional to D^{*2}). Some care must be used to insure that the base-pressure model is valid for the base area under consideration.

For purposes of comparison, the optimum nozzles predicted by Scofield and Hoffman[1] for the specified operating conditions were placed in a vehicle of the same external diameter as considered in the present analysis. In cases involving relatively small-diameter vehicles operating at high r_o, the predicted optimum nozzle required an exit area larger than the proposed vehicle cross sectional area. This inconsistency cannot exist in the present analysis, since vehicle diameter is one of the imposed physical constraints in the optimization. It should be pointed out here that the truncated perfect and Rao nozzles compared by Scofield and Hoffman[1] can also be compared to the results of the present analysis in a manner similar to the above. Figures 3 and 4 present the thrust improvement obtained by the present analysis over nozzles designed by the Scofield-Hoffman technique. These results are indicative of the performance increases obtainable using the proposed design technique. The magnitude of these improvements will depend upon the base pressure model employed. However, these results do exhibit trends which should be useful in designing conical thrust nozzles when an annular base region is present.

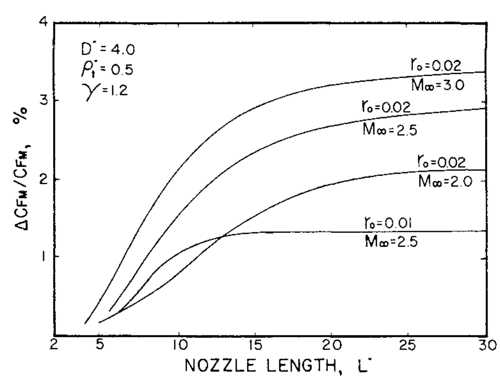

Fig. 4 Effect of flight conditions on performance.

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Radiant Heat Transfer between Simply Arranged Surfaces with Direction Dependent Properties

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RECENT investigations have shown that there is considerable discrepancy between the radiant heat-transfer predictions based on the diffuse, specular, and diffuse + specular analyses and experimental data[1,2] or more realistic direction-dependent property models.[1,3] The choice of the model for the radiation characteristics of surfaces is most critical for highly reflecting surfaces approaching the optically smooth condition.[1,2] The level of detail in the models which is needed in radiant heat transfer calculations has not been clearly established but appears to be most important when predicting local radiant energy quantities such as the flux, radiosity or irradiation.[3] This Note compares results for radiant heat-transfer calculations based on a directional emission and reflection model[1,3] with available experimental data.[2,4] A limited number of calculations based on the model were carried out and found to be promising.[1] The model appears to be a good compromise between realistic level of detail (where needed) and computational effort and is explored here in more detail. Results are reported for two configurations: 1) two parallel, infinitely long plates, and 2) two parallel, infinitely long plates separated on one edge by either a diffuse or a specular adiabatic plate (Fig. 1).

Model and Method of Solution

It is assumed that the geometric optics theory is valid for radiant heat transfer and that the surfaces are separated by a nonparticipating medium having an index of refraction of unity. The radiation surface characteristics and temperatures or heat transfer rates at the surfaces are assumed to be given.

Presented as Paper 69-624 at the AIAA 4th Thermophysics Conference, San Francisco, Calif., June 16–18, 1969; submitted December 12, 1969. Work supported by the NASA Manned Spacecraft Center under Contract NAS 9-8118.

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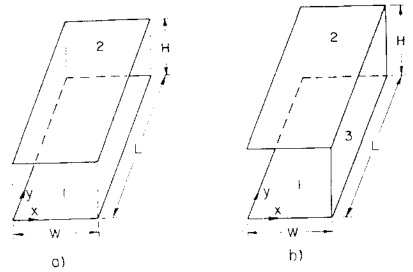

Fig. 1 Configurations analyzed: a) parallel plate system, b) perpendicular plate system.

The directional emission and reflection model,[1,3] for which the diffuse (ρ^d) and specular (ρ^s) components of reflectivity depend on incident direction, θ', and wavelength, λ, i.e.,

$$\rho(\theta', \lambda) = \rho^d(\theta', \lambda) + \rho^s(\theta',\lambda) \tag{1}$$

is adopted. The dependence of the radiation characteristics on direction is predicted from Fresnel's equations.[5]

The directional and spectral dependence of radiation surface properties are predicted for an ideal engineering surface. The optical constants are calculated by Drude's relation.[6] Bennett et al.[7] have demonstrated that Drude's theory is in excellent agreement with the experimental data for high conductivity metals. For high alloy steels, the theory predicts reasonably well the qualitative dependence of reflectance on wavelength. For the purpose of this study the choice of Drude's relation to predict the optical constants is considered reasonable.

The specular components of directional reflectivity, $\rho^s(\theta',\lambda)$, are approximated by[8,9]

$$\rho^s(\theta',\lambda) \simeq \rho(\theta',\lambda) \exp\{-4\pi[(\sigma_0/\lambda)\cos\theta']^2\} \tag{2}$$

and the specular component of the hemispherical reflectivity is given by

$$\rho^s(\lambda) = \frac{1}{\pi}\int_0^{2\pi}\int_0^{\pi/2} \rho^s(\theta',\lambda)\cos\theta' \sin\theta' \, d\theta' \, d\phi' \tag{3}$$

where $\rho(\theta',\lambda)$ is the directional reflectivity of an optically smooth surface and σ_0 is the rms optical roughness. Use of the approximate Eq. (2) is partly justified by the difficulty in accounting rigorously for the finite conductivity in the theory.[8]

The calculations were performed on a digital computer using the Monte Carlo method. The simulation of radiant heat transfer by the method is detailed elsewhere[3,10]; suffice it to say that histories of 10,000 energy bundles were traced for each cruve. The computations were performed on the

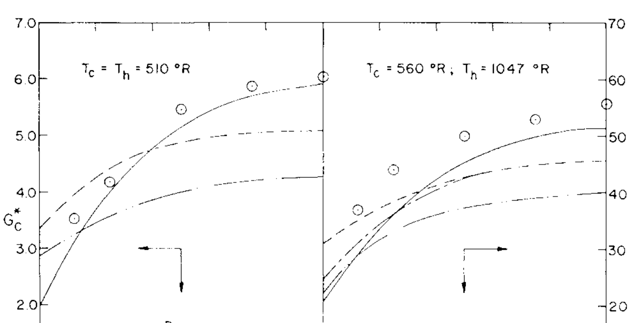

Fig. 2 Comparison between different models and experimental data[2] for polished stainless steel (Type 303), σ_m = 0.28; parallel plate system, γ = 0.125 ($\xi \equiv x/W$; $G_c^* = G_c/\epsilon_c\sigma T_c^4$.

gray basis, and the optical roughness, σ_0, was replaced by the rms mechanical roughness, σ_m. This is only approximate, as it is well known that $\sigma_0 > \sigma_m$. The wavelength λ was replaced by a mean wavelength λ_m, which was evaluated at the peak of the spectral blackbody emissive power curve corresponding to the particular surface temperature. This approach is somewhat arbitrary, but in view of the fact that $\sigma_m < \sigma_0$ and $\lambda_{max} < \lambda_m$, the ratio σ_m/λ_{max} is expected to be a reasonable approximation for σ_0/λ_m.

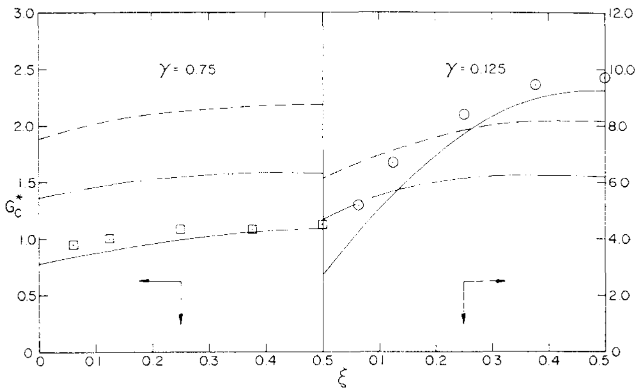

Fig. 3 Comparison between different models and experimental data[2] for smooth electroplated gold, σ_m = 0.075; parallel plate system, $T_c = T_h$ = 510°R.

Results and Discussion

The choices of geometry, (Fig. 1), surface materials, and temperature conditions have been guided by the availability of experimental data.[2,4] For comparison with the present model [coded D + S (DIRECTIONAL) in the figures], the predictions based on the diffuse, (D), specular (S), and diffuse + specular (D + S) constant-property models are included in the figures (see key in Fig. 2). The results are presented in terms of local ($\xi = x/W$) irradiation values at the cold surface, G_c^* (nondimensionalized with respect to the emissive power of the cold surface), because this is the quantity which has been measured experimentally.

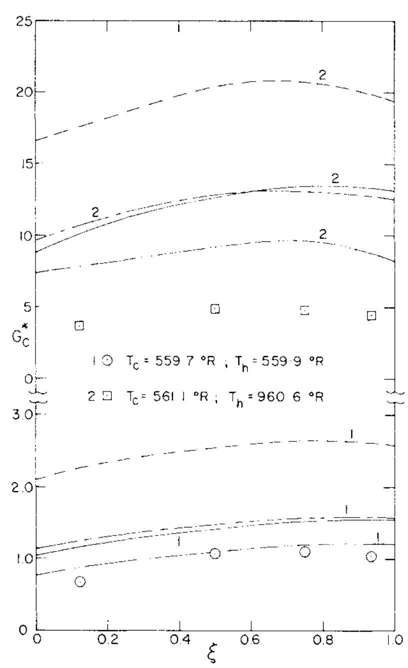

Fig. 4 Comparison between different models and experimental data[4] for rough electroplated gold, σ_m = 0.55; perpendicular plate system with adiabatic, diffuse, perfectly reflecting surface 3, γ = 1.0.

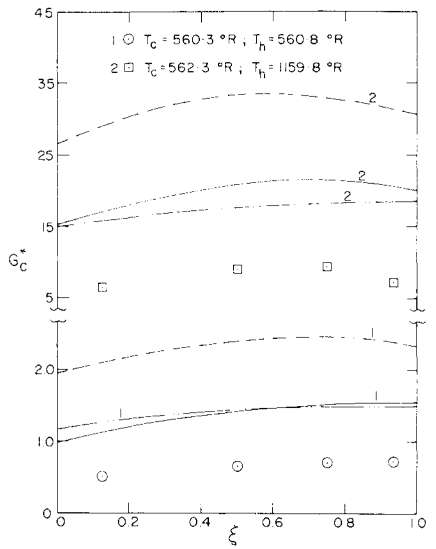

Fig. 5 Comparison between different models and experimental data[4] for smooth electroplated gold, σ_m = 0.075; perpendicular plate system with adiabatic, diffuse, perfectly reflecting surface, 3, γ = 1.0.

The results for the parallel-plate geometry are presented in Figs. 2 and 3. The curves in all figures have been faired through the computed points using the Monte Carlo method. The Monte Carlo results for the D, S, and $D + S$ models are in good agreement with the results obtained by solving numerically the governing integral equations.[1,2] The D + S curves are not shown in Figs. 2 and 3, because for the surfaces and temperatures considered the specular component of reflectivity predominated, and the D + S predictions are practically identical to those of the specular (S) model. Surprisingly, the experimental data (circles and squares) agree best with the predictions based on the diffuse analysis even for a smooth electroplated gold surface, Fig. 3, for which $\rho^s/\rho \approx 1.0$. The specular model overpredicts the local irradiation by a factor of two for the larger spacing-to-width ratio, $\gamma = H/W = 0.75$.

In Figs. 4 and 5 the predictions of the different models are compared with experimental data for the case when the adiabatic, perfectly reflecting surface 3 is diffuse (σ_m = 8.9 μ). The predictions of the present D + S DIRECTIONAL model are in general, in best agreement with the experimental results. For surfaces approaching the optically smooth condition (Fig. 5), the predictions based on the specular analysis are in some instances higher than the experimental data by about a factor of five. The predictions based on the diffuse model are in better agreement with the experimental data than the specular model, even for a smooth electroplated gold surface. It is not possible, however, to draw any general conclusions from the limited range of parameters presented in the figures.

Conclusions

The results obtained show the importance of directional effects; however, the directional property model considered is not completely satisfactory even though it is more detailed than the diffuse, specular, and diffuse + specular models. There are discrepancies between the predictions and the experimental data. These discrepancies are considerably greater than the estimated accuracy of the data.[2,4]

Perhaps the main value of this study is that it emphasizes our incomplete understanding of, and inability to predict accurately, the radiation characteristics of real surfaces, as well as the need for more experimental radiant heat-transfer data, particularly on a spectral basis, in order to separate the directional from the spectral effects.

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